Analytical approach to the drift of the tips of spiral waves in the complex Ginzburg-Landau equation

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In this paper, we investigate the motion of spiral waves in the complex Ginzburg-Landau equation (CGLE) analytically and numerically. We find that the tip of the spiral wave drifts primarily in the direction of the electric field and there is a smaller component of the drift that is perpendicular to the field when a uniform field is applied to the system. The velocity of the tip is uniform and its component along the electric field is equal to the strength of the field. When the CGLE system is driven by white noise, a diffusion law for the vortex core of the spiral wave is derived at long time explicitly. The diffusion constant is found to be $D=T/C^2$, in which T is the noise strength and C is the core asymptotic factor of the spiral wave. When the external force is a simple oscillation we find that the tip of the spiral wave drifts if the frequency of the external force is the same as that of the system. Our analytical results are verified using numerical simulations.

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I. INTRODUCTION

Spiral waves are significant patterns that are ubiquitous in many systems of physics, chemistry, materials, and biology [1]. They occur in the reaction-diffusion media [2], certain regions of fluid flows [3], the CO oxidation on platinum surfaces [4], the aggregating slime-mold cell [5], and the contraction of heart muscles [6]. Meandering and drifting spirals have been observed experimentally, for instance, in the Belousov-Zhabotinsky (BZ) reaction [7–10], the oxidation of carbon monoxide on platinum surfaces [11], and during fibrillations in cardiac tissue [12]. The center of the spiral wave is observed to drift in the BZ reaction when a uniform electric field is applied in the reaction dish [13]. In order to have an insight into the motion of spiral waves, the drift of spiral waves has been studied theoretically in many different systems [14–21]. However, to the best of our knowledge, almost all published studies are based on numerical simulations, to date, there have been no analytical investigations. Due to this lack of analytical study, some observations of spiral waves, such as the component of the spiral wave drift that is perpendicular to the field when an electric field is applied to an oscillatory reaction-diffusion system [13], have not been fully understood.

Recently, a velocity formula for the spiral wave in a reaction-diffusion system was derived in a paper under the nondeformation approximation of the spiral wave core [22]. We develop this study for the complex Ginzburg-Landau equation (CGLE) in this paper, which describes a vast variety of phenomena from nonlinear chemical wave to second-order phase transitions, from superconductivity, superfluidity, and Bose-Einstein condensation to liquid crystals [3,23–26]. In the oscillatory regime, sufficiently close to the onset of oscillation, the dynamics of the BZ reaction are modeled by the CGLE [24]. In this paper, the motion of spiral waves is investigated in the presence of an external force, noise, and time-dependent periodic background. After deriving an analytical expression for the velocity for each case we compare our analytical results with corresponding numerical simulations.

This paper is organized as follows: An analytical expression for the velocity of the spiral tip of CGLE is derived for a general external perturbation in Sec. II. When a uniform external vector field (such as an electric field) is applied, the motion of the spiral tip is investigated analytically in Sec. III. The corresponding numerical simulation is also executed to examine our analytical results. We find that the tip of the spiral wave drifts primarily in the direction of the electric field, but there is also a small component of the drift that is perpendicular to the field, which agrees well with observations [13]. The velocity of the tip of the spiral wave is equal to the strength of the external field and the motion of the tip is along a straight line with uniform velocity. In Sec. IV, using the analytical results of Sec. II, the behavior of the tip of the spiral wave is explored when there is a weak additive broadband noise in the CGLE system and a diffusion law for the tip of the spiral wave at long times is derived analytically. It is found that the diffusion constant is proportional to the noise strength and inversely proportional to the square of the core asymptotic factor of the spiral wave. In Sec. V, the motion of the tip of a spiral wave is studied in the case of a periodic force being applied in the CGLE system. We find that the tip of the spiral wave drifts if the frequency of the driving force is equal to that of the CGLE system. Finally, a summary and a closing remark are given in Sec. VI.

II. VELOCITY OF THE SPIRAL WAVE TIP

It is well known that the usual CGLE is given by [23]

$$\partial_t A = A + (1+ib)\nabla^2 A - (1+ic)|A|^2 A, \tag{1}$$

where $A \equiv A_1 + iA_2$ is a complex function of time t and space (x,y), the real parameters b and c characterize linear and nonlinear dispersion and ∇^2 stands for the Laplacian operator. Spiral waves are observed if $b \neq c$. A single-armed spiral wave solution to Eq. (1) is given by [27]

$$A_s(\mathbf{x},t)=F(r)\exp\{i[\sigma\theta+\psi(r)-\omega t]\}, \tag{2}$$

in which $r=|\mathbf{x}-\mathbf{x}_0|$ and θ is the polar angle measured from the vortex core centered at $\mathbf{x}_0$, and $\sigma=\pm1$ is the topological charge of the spiral wave. Far away from the core solution (2) approaches a plane wave with $F\simeq\sqrt{1-k_0^2}$ and $\psi(r)\simeq k_0 r$, where the asymptotic wave number k_0 depends on the parameters b and c. The real functions $F(r)$ and $\psi(r)$ have the following asymptotic behavior: $F\simeq\psi'\simeq r$ as $r\to0$.

When an arbitrary external perturbation influences the system, Eq. (1) becomes as follows:

$$\partial_t A=A+(1+ib)\nabla^2A-(1+ic)|A|^2A+\Gamma. \tag{3}$$

The external perturbation $\Gamma\equiv\alpha+i\beta$ may or may not be dependent on A. For example, when an electric field is present one has $\Gamma=\mathbf{E}\cdot\boldsymbol{\nabla}A$, where $\mathbf{E}$ stands for the vector parameters of the electric field [14]. Assuming that Γ induces a drift of the spiral wave tip with velocity $\mathbf{V}(t)=\mathbf{e}_xV_x(t)+\mathbf{e}_yV_y(t)$, $\mathbf{e}_x$ and $\mathbf{e}_y$ are two unit vectors along the x axis and y axis, respectively, we can rewrite Eq. (3) in the comoving coodinate system as

$$\partial_t A=A+(1+ib)\nabla'^2A-(1+ic)|A|^2A+\mathbf{V}\cdot\boldsymbol{\nabla}'A+\Gamma, \tag{4}$$

where $\boldsymbol{\nabla}'$ is the gradient operator in the comoving coordinate system.

We assume that the deformation of the spiral wave in the core is small enough to be neglected for small Γ. Then Eq. (1) holds true in the comoving coordinate system. From Eqs. (1) and (4) we have

$$V_x\partial'_xA_1+V_y\partial'_yA_1=\alpha,\quad V_x\partial'_xA_2+V_y\partial'_yA_2=\beta.$$

Since $\boldsymbol{\nabla}'=\boldsymbol{\nabla}$ for homogeneous motion, the velocity $\mathbf{V}$ can be expressed as follows:

$$V_x=\frac{\alpha\partial_yA_2-\beta\partial_yA_1}{\partial_xA_1\partial_yA_2-\partial_xA_2\partial_yA_1},$$

$$V_y=-\frac{\alpha\partial_xA_2-\beta\partial_xA_1}{\partial_xA_1\partial_yA_2-\partial_xA_2\partial_yA_1}. \tag{5}$$

Making use of the spiral wave solution (1) and its asymptotic behavior, the velocity of the tip of the spiral wave can be determined completely. A similar formula has been derived in the modified FitzHugh-Nagumo model and the corresponding drift of the spiral wave studied in Ref. [22].

Using the single-armed spiral wave solution (2) in the limit $r\to0$, $F\simeq\psi'\sim r$, we can write the velocity of the tip of the spiral wave from Eq. (5) in the form

$$V_x=\frac{1}{2C}[\alpha\cos(\omega t)-\beta\sin(\omega t)],$$

$$V_y=\frac{1}{2C}[-\alpha\sin(\omega t)+\beta\cos(\omega t)], \tag{6}$$

where C is the constant in the limit $F(r\to0)=Cr$, which represents the rate of change of the spiral wave amplitude with respect to the distance from the vortex core. We call C the core asymptotic factor of the spiral wave.

III. MOTION OF SPIRAL WAVES IN THE PRESENCE OF UNIFORM ELECTRIC FIELDS

Following Ref. [14], a reaction-diffusion system applied in a uniform external field is modeled by the CGLE with an additional term:

$$\partial_t A=A+(1+ib)\nabla^2A-(1+ic)|A|^2A+\mathbf{E}\cdot\boldsymbol{\nabla}A. \tag{7}$$

The external field is chosen in the form $\mathbf{E}=\mathbf{e}_x(\gamma+i\delta)$ for simplicity, where γ and δ are two real constants. The external perturbation becomes

$$\Gamma=(\gamma\partial_xA_1-\delta\partial_xA_2)+i(\gamma\partial_xA_2+\delta\partial_xA_1). \tag{8}$$

Using Eqs. (6) and (8), we derive the drift velocity of the spiral wave as follows:

$$V_x=\gamma-\delta\frac{(\partial_xA_1)^2+(\partial_yA_2)^2}{\partial_xA_1\partial_yA_2-\partial_xA_2\partial_yA_1},$$

$$V_y=\delta\frac{(\partial_xA_1)^2+(\partial_yA_2)^2}{\partial_xA_1\partial_yA_2-\partial_xA_2\partial_yA_1}. \tag{9}$$

Note that the above formula is obtained under the condition that there is no deformation of the spiral wave. From the asymptotic behavior of solution (2) under $r\to0$, we can derive the velocity of the spiral wave as follows:

$$V_x=\gamma+\frac{\delta}{2\sigma}\{1+\cos2[\psi(0)-\omega t]\},$$

$$V_y=\frac{\delta}{2\sigma}[1+\cos2(\psi(0)-\omega t)]. \tag{10}$$

We see that the velocity is proportional to the parameters of the external field and depends on the terms $\cos2(\psi(0)-\omega t)$.

If the external field is electric, then all the components of $\mathbf{E}$ are real [14]. It is easy to see that the velocity is only along the x axis and the magnitude of the velocity is equal to the magnitude of the electric field, i.e., $V_x=\gamma$ and $V_y=0$. We simulated Eq. (7) numerically with the Runge-Kutta algorithms. Calculations were done in a two-dimensional 356×356 array with a time step $h_t=0.05$. Von Neumann's "no flux" boundary condition was imposed on the boundary of the medium. In the computation we used the following parameters: $\gamma=0.4$ and $\delta=0$ numerically. We used the values $b=-1.0$ and $c=0.5$. The electric field was applied for $\triangle t=800$ after the spiral wave had formed. Our numerical simulation showed that the tip of the spiral wave drifts primarily in the direction of the electric field. Figure 1 gives the orbit of the tip of the spiral wave. We see that there is a smaller component of the drift that is perpendicular to the electric field. It can be seen that the rate of the x-component drift to the y-component drift is about 4.4%. The perpendicular drift

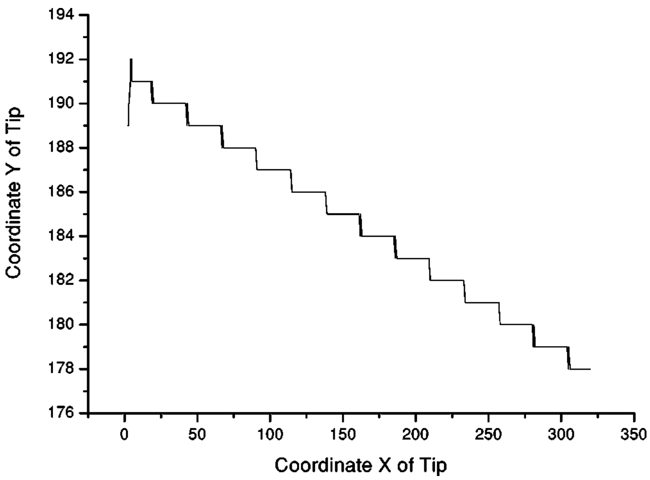

FIG. 1. When a uniform electric field is applied, the trace of the spiral wave tip is a straight line. This figure is plotted with $b=-1.0$, $c=0.5$, and $\gamma=0.4$. The tip of the spiral wave drifts primarily in the direction of the electric field, i.e., the x axis. There is also a smaller component of the drift that is perpendicular to the field. The ratio of the x-component drift to the y-component drift is about 4.4%.

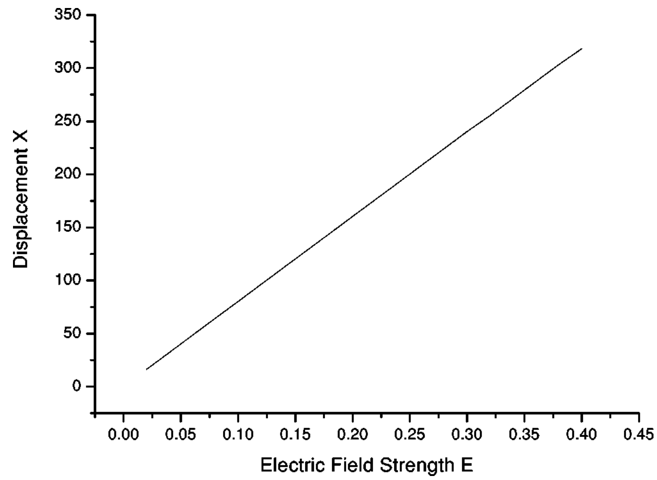

FIG. 3. The x-component displacement of the spiral wave tip depends on the strength of the electric field for a fixed time of $\triangle t=800$. In this figure, the constants b and c have the same values as in Figs. 1 and 2. The strength of the electric field ranges from 0.02 to 0.4 with a step of 0.02. We determine the slope of the straight line to be 800. Using $\triangle t=800$, we derive the x-component velocity to be equal to the strength of the electric field.

of the spiral wave tip has been observed in the BZ spiral wave [13], but was not confirmed in previous theoretical investigation of the reaction-diffusion system in the oscillatory regime [14].

We simulated Eq. (7) with $b=-1.0$, $c=0.5$, and $\delta=0$ for $\gamma=0.1$, 0.3, 0.4 and applied the electric field for $\triangle t=800$ after a single-armed spiral wave had formed. Figure 2 shows the x-component displacement of the tip of the spiral wave vs time. It can be seen that the relationship between the displacement and time is linear. This means that the velocity of the spiral tip is uniform for a fixed strength of the electric field. The slope of the straight line is the velocity of the tip and increases when the strength of the electric field is increased. We calculated the slopes of the straight lines as $-0.1, -0.3, -0.4$ and found that the velocity of the spiral tip was equal to the electric field strength.

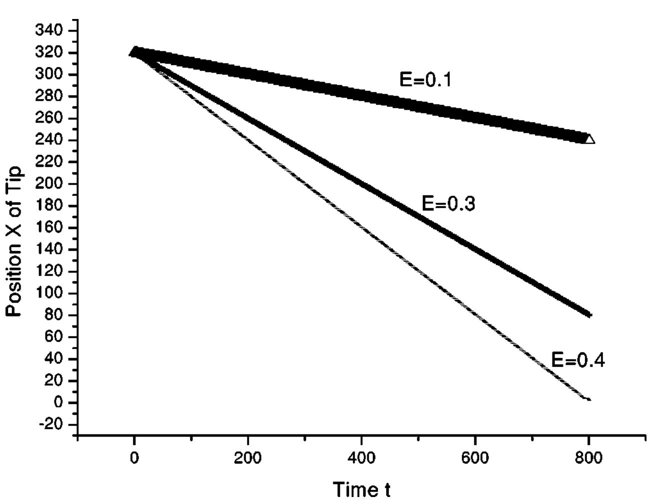

FIG. 2. The x-component displacement of the spiral wave tip varies with time when the uniform electric fields with strength $\gamma=0.1$, 0.3, 0.4 are applied, respectively. We plot this graph with $b=-1.0$ and $c=0.5$. The corresponding x-component velocities are equal to the strength of the electric field, i.e., 0.1, 0.3, 0.4.

Figure 3 demonstrates the relationship between the x-component displacement of the spiral wave tip and the strength of the electric field for the fixed time $\triangle t=800$. The strength of the electric field ranges from 0.02 to 0.4 with a step of 0.02. The constants in (7) were set at $b=-1.0$, $c=0.5$, and $\delta=0$. We see that the relationship between the x-component displacement and the electric field strength is linear. In other words, the x-component displacement is proportional to the strength of the electric field. Since the velocity of the spiral wave tip is uniform, which has been shown in Fig. 2, it is also proportional to the strength of the electric field. This agrees with the conclusions obtained from the last paragraph. The slope of the straight line in Fig. 3 is 800. Using the definition of velocity, we can also conclude that the x-component velocity of the spiral wave tip is equal to the strength of the electric field.

IV. RESPONSE OF SPIRAL WAVES TO ADDITIVE NOISE

External noise perturbations often appear in spiral waves experiments. Aranson *et al.* investigated spiral motion in a noisy CGLE using a linear response assumption [16]. They showed that a spiral core driven by white noise has a finite mobility and performs Brownian motion. In this section we analytically investigate the motion of the tip of spiral waves in the CGLE. The CGLE with weak additive broadband noise η can be expressed as

$$\partial_t A = A + (1+ib)\nabla^2 A - (1+ic)|A|^2 A + \eta,$$

in which η is generally a complex function of space coordinate and time.

We consider the special case where $\eta(\mathbf{x},t)$ takes the form of weak, uncorrelated white noise with zero mean and correlators,

$$\langle \eta_\mu(\mathbf{x},t)\,\eta_\nu(\mathbf{x}',t')\rangle = 2T\delta_{\mu\nu}\delta(\mathbf{x}-\mathbf{x}')\,\delta(t-t'), \tag{11}$$

where μ andν specify real and imaginary parts of η and T characterizes the noise strength. Using the spiral wave solution (2) and its asymptotic behavior, we derive a velocity expression for the tip of spiral waves in the form

$$V_x = \frac{1}{2C}[\,\eta_\mu \cos(\omega t) - \eta_\nu \sin(\omega t)],$$

$$V_y = \frac{1}{2C}[-\eta_\mu \sin(\omega t) + \eta_\nu \cos(\omega t)], \tag{12}$$

where ω is the angular frequency of the unperturbed spiral wave. Because η_μ and η_ν are random, it can be seen that the spiral wave performs Brownian motion.

From the velocity (12), we determine the coordinates of the spiral wave tip as follows:

$$x = x(t=0) + \frac{1}{2C}\int_0^t [\,\eta_\mu \cos(\omega\tau) - \eta_\nu \sin(\omega\tau)]d\tau,$$

$$y = y(t=0) + \frac{1}{2C}\int_0^t [-\eta_\mu \sin(\omega\tau) + \eta_\nu \cos(\omega\tau)]d\tau.$$

Using the white noise correlators (11), we obtain a diffusion law for the vortex core at long times:

$$\langle r^2\rangle = \langle [x - x(t=0)]^2\rangle + \langle [y - y(t=0)]^2\rangle = Dt. \tag{13}$$

In Eq. (13), the diffusion constant is $D = T/C^2$, where T is the noise strength in Eq. (11) and C is the core asymptotic factor that is defined in terms of the asymptotic behavior in Sec. II. It can be seen that our analytical result (13) is exactly the same as the conclusion in Ref. [16], but we determine the diffusion constant completely analytically.

V. MOTION OF SPIRAL WAVES WITH A TIME-DEPENDENT PERIODIC EXTERNAL FORCE

When a time-dependent periodic external force is applied to the CGLE system, its dynamics are governed by the following equation:

$$\partial_t A = A + (1+ib)\nabla^2 A - (1+ic)|A|^2 A + \eta_0 \exp(-i\Omega t), \tag{14}$$

where η_0 is the amplitude of the periodic force and Ω stands for its frequency. Using the general expression for the perturbation drift velocity, Eq. (5), we derive the velocity of the spiral wave tip driven by the time-dependent periodic external force as

$$V_x = \frac{\eta_0}{2C}\cos[(\omega-\Omega)t - \psi(0)],$$

$$V_y = \frac{\sigma\eta_0}{2C}\sin[(\omega-\Omega)t - \psi(0)], \tag{15}$$

in which ω is the angular frequency of the unperturbed spiral wave and C is the core asymptotic factor defined in Sec. II. From Eq. (15), we see that V_y is dependent on the topological charge σ of the spiral wave. Generally, the tip of a spiral wave oscillates in both the x and y directions, in which the oscillatory frequency is equal to $\omega-\Omega$. Equations (15) show that the amplitudes of the tip in both the x and y directions are the same. Therefore, the spiral wave tip drifts along a circle with radius

$$R = \frac{\eta_0}{2C(\omega-\Omega)},$$

in which the frequency of the time-dependent external force must not be equal to that of the unperturbed spiral wave. We find that the less the difference of the frequency, the larger the radius of the circle.

If the frequency of the time-dependent external force is equal to that of the unperturbed spiral wave in the CGLE, i.e., $\Omega=\omega$, Eq. (15) becomes

$$V_x = \frac{\eta_0}{2C}\cos\psi(0), \quad V_y = -\frac{\sigma\eta_0}{2C}\sin\psi(0). \tag{16}$$

We find that the tip of the spiral wave drifts along a straight line with uniform velocity when a time-dependent periodic force is applied to the system. The direction of the straight line is defined by the $r\rightarrow 0$ asymptotic behavior of the phase function $\psi(r)$ in the general expression of the spiral wave solution (2). It is also dependent on the topological charge of

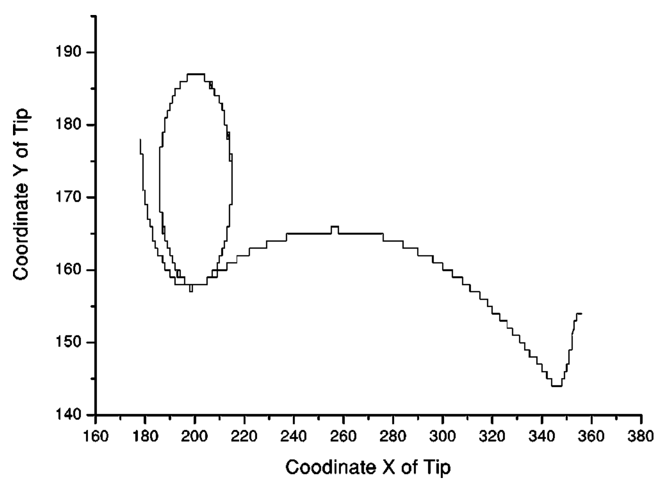

FIG. 4. When the frequency of an external periodic force applied to the system is equal to that of the unperturbed spiral wave, the tip of the spiral wave drifts accordingly. In this figure, $b=0$, $c=0.7$, and $\eta_0=0.008$. The unperturbed single-armed spiral wave is formed by making use of $t=1200$. Then, the external force is applied and remains for $\triangle t=9000$. The curve shows the trajectory of the tip of the spiral wave during the application of the time-dependent periodical external force.

the spiral wave. We see that the total velocity is proportional to the amplitude of the external force and inversely proportional to the core asymptotic factor defined in Sec. II.

In order to verify our analytical result, we simulate Eq. (14) numerically with $b=0$ and $c=0.7$ in a 356×356 array. The time step is 0.05. The amplitude of the force, η_0, is taken as 0.008. Using the approximation formula [27],

$$\omega = c(1-k^2), \quad k \simeq -c^{-1}\exp(-\pi/2|c|)$$

for $b=0$ and $|c|\ll 1$, we obtain the approximate frequency of the unperturbed spiral wave as $\omega\approx 0.683\,938\,2$. The frequency of the time-dependent periodic external force is finally determined to be $\Omega=0.679\,138\,2$ by means of a numerical test. After a single-armed spiral wave is formed, the external force is applied for $\triangle t=9000$. Figure 4 shows the numerical result of the trace of the drift of the spiral wave tip in the case of the time-dependent periodic external force of frequency Ω. It can be seen that, indeed, the spiral wave tip drifts when the time-dependent periodic external force resonates with the CGLE system.

VI. CONCLUSION AND REMARKS

In this paper, we have investigated the motion of the tip of spiral waves in the CGLE analytically. We found that the tip of the spiral waves drifts primarily in the direction of the electric field with a small component of the drift perpendicular to the field, when a uniform electric field is applied to the system. This agrees well with experimental observations [13]. The spiral wave tip velocity in the direction of the electric field was equal to the strength of the field. The velocity that is perpendicular to the field is also uniform. A diffusion law is obtained if weak additive broadband noise exists in the system. The diffusion constant was determined to be proportional to the noise strength and inversely proportional to the core asymptotic factor of the spiral wave. When a periodic, time-dependent external force with frequency equal to that of the unperturbed spiral wave is applied to the CGLE system, the drift of the spiral wave appears.

It must be pointed out that in our analytical study, the deformation of the tip of the spiral wave is neglected and any effects caused by the deformation of the spiral wave are not included in this paper. The smaller component of the drift that is perpendicular to the uniform electric field may be due to deformations of the spiral wave.

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